The Mathematical Association of Victoria

Further Mathematics

2006 Written Examinations

Solutions

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2006 Further Mathematics
Written Examination 2 (Analysis task)
Suggested answers and solutions

Core

Question 1

a **18-month-old boys**

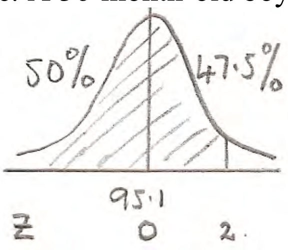
Input Column 1 from Table 1 into calculator:
STAT
CALC
Enter 1 : 1-Var Stats for your list.

```
1-Var Stats
 x̄=82.67142857
 Σx=1157.4
 Σx²=95875.76
 Sx=3.841559894
 σx=3.70181974
↓n=14
```

The standard deviation (S_x) for 18 months is 3.8 (correct to one decimal place).

b Z-score = $\dfrac{x-\bar{x}}{s_x} = \dfrac{83.1-89.3}{4.5} = -1.4$ (correct to one decimal place)

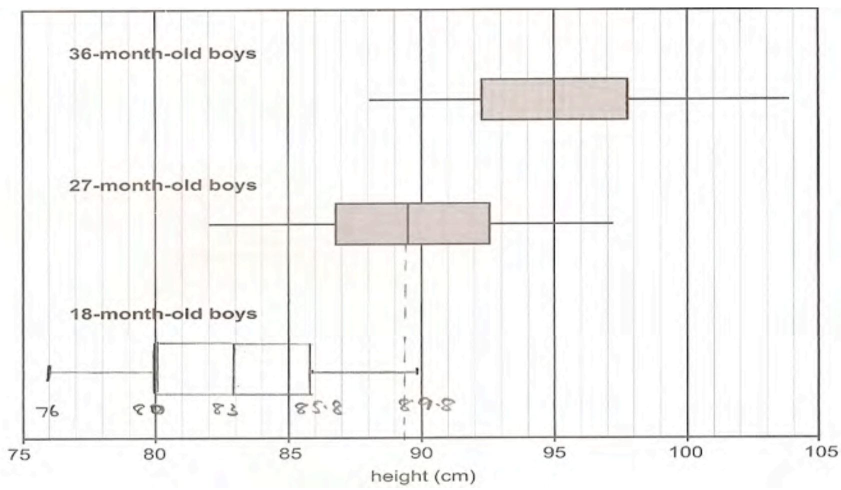
c. A 36-month-old boy has a standardised height Z value of 2.



Approximately 97.5% of 36-month-old boys will be shorter than this child.

d. 18-month-old boys five number summary

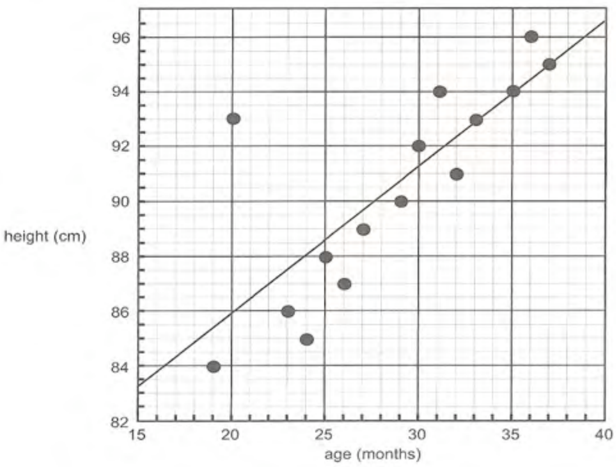
```
1-Var Stats
↑n=14
 minX=76
 Q1=80
 Med=83
 Q3=85.8
 maxX=89.8
```



e. From the boxplot for 27-month-old boys, the median height is 89.5cm.

f. The median of heights for 18-month-old, 27-month-old and 36-month-old boys is 83cm, 89.8cm and 95cm respectively. The median is increasing, suggesting that height and age are positively related.

Question 2



The least squares regression line is Height $= 75.4 + 0.53 \times$ age

The correlation coefficient is $r = 0.7541$

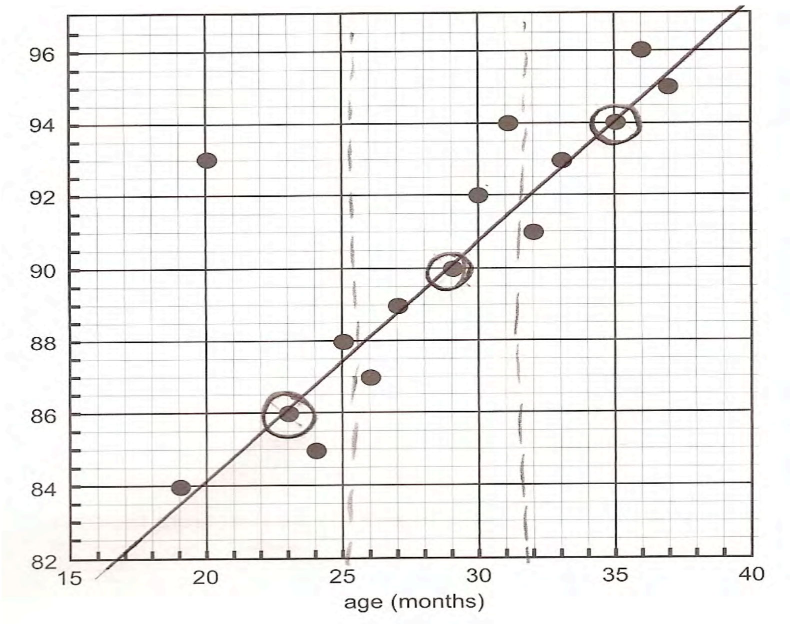
a. On average, the height of a boy increases by **0.53** cm for each one-month increase in age.

b. i The coefficient of determination $r^2 = (0.7541)^2 = 0.5687 = 56.9\%$ (correct to one decimal place)

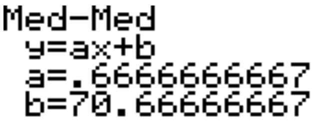
ii 56.9% of the variation in height can be explained by the variation in age.

Question 3

a.



b. Input data into 2 lists L_1, L_2 and do a 3 **Med-Med**. Line



The three median line is: height = $70.7 + 0.7 \times$age (correct to 1 decimal place)

c. As there is an outlier at (20, 93), the three median line is the preferable model to use as the least squares regression line is significantly affected by outliers.

Module 1: Number patterns and applications

Question 1

a. At the end of the second day there are $48000 - 2(3000) = 42000$ kg of fruit

b. The value of d is -3000.

c. If all the fruit is picked from the trees then: $-3000 \times n + 48000 = 0$
So $n = 16$ days.

Question 2

a. $r = \dfrac{t_2}{t_1} = \dfrac{500}{625} = 0.8$ and $r = \dfrac{t_3}{t_2} = \dfrac{400}{500} = 0.8$

So $r = 0.8$, as required.

b. $a = 625 \quad r = 0.8$ and $t_n = ar^{n-1}$

$t_5 = ar^{5-1} = 625 \times 0.8^4 = 256$

The gardeners will work 256 hours in the fifth month.

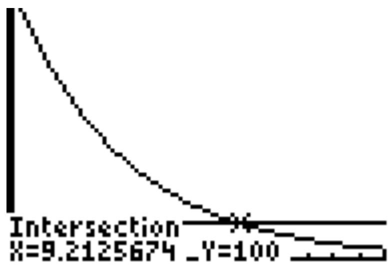
c. The number of hours that the gardeners will work in the nth month after planting is $H_n = 625 \times 0.8^{n-1}$

d. $t_6 = 204.8$ hours and $t_7 = 163.84$ hours

$204.8 - 163.84 = 40.96$

The gardeners work 41 hours more (to the nearest hour).

e. Solve $100 = 625 \times 0.8^{n-1}$, using the graphics calculator.



Therefore the gardeners work less than 100 hours during the 10th month.

f. In the next nine months, i.e. months 3 to 12, the gardeners work $S_{12} - S_3$.

$= 2910.25 - 1525$

$= 1385.25$

Answer: 1385 hours (to the nearest hour).

Question 3

a. The volume of water, V_n, in the tank on the morning of the nth day is modelled by the difference equation $V_{n+1} = rV_n + d$ where $V_1 = 45000$ litres

As 10% of the volume is used, $r = 0.9$

As 2000 litres is added to the tank, $d = 2000$

b. $V_{n+1} = 0.9V_n + 2000$

$V_1 = 45000$ litres

$V_2 = 0.9V_1 + 2000 = 0.9(45000) + 2000 = 42500$

$V_3 = 0.9V_2 + 2000 = 0.9(42500) + 2000 = 40250$

$V_4 = 0.9V_3 + 2000 = 0.9(40250) + 2000 = 38225$

There is 38225 litres of water in the tank on the morning of the fourth day.

c. To find when the tank will first be below 30000 litres, solve the difference equation:

```
Plot1 Plot2 Plot3
 nMin=1
\u(n)■0.9u(n-1)+
2000
 u(nMin)■{45000}
\v(n)=
 v(nMin)=
\w(n)=
```

.

n	u(n)	
4	38225	
5	36403	
6	34762	
7	33286	
8	31957	
9	30762	
10	29686	

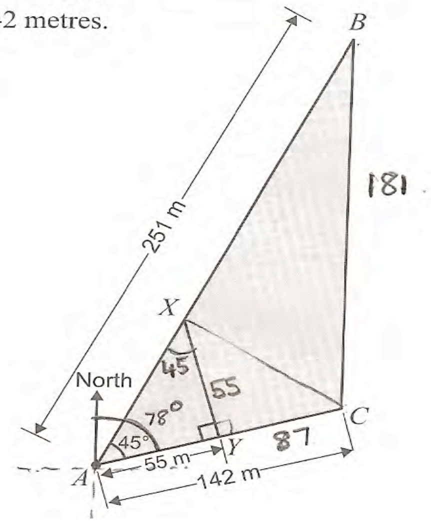
n=10

The tank will first be below 30000 litres during the 10th day.

d. There will be 20000 litres in the tank each morning. It drops 10% to 18000 litres during the afternoon but then 2000 litres is added each evening bringing it back to 20000.

Module 2: Geometry and trigonometry
Question 1

A farmer owns a flat allotment of land in the shape of triangle ABC shown below.
Boundary AB is 251 metres.
Boundary AC is 142 metres.
Angle BAC is 45°.



a. $\angle AXY = 180^0 - (45+90)^0 = 45^0$

b. $\cos 45^0 = \dfrac{55}{AX}$

$$AX = \frac{55}{\cos 45^0}$$
$$= 77.8 \quad \text{(correct to 1 decimal place)}$$

c. From the diagram above, the bearing of B from A is $78° - 45° = 033°T$

d. $\tan 45^0 = \dfrac{XY}{55}$

$$XY = 55\tan 45^0$$
$$= 55$$
$$XC = \sqrt{55^2 + 87^2} = 102.9 \text{ (correct to 1 decimal place)}$$

e.

$$\text{Area of } \Delta\text{ABC} = \frac{1}{2} \times 251 \times 142 \times \sin 45^0$$
$$= 12601.34$$
$$\approx 12601 \text{ m}^2 \text{ (correct to nearest square metre)}$$

Alternatively, Heron's Rule can be used to find the area of the triangle.
Using the Cosine rule, $BC = \sqrt{251^2 + 142^2 - (2 \times 251 \times 142\cos 45^0)} = 181 \text{ m}$

$$s = \frac{1}{2}(a + b + c)$$
$$= \frac{1}{2}(251 + 142 + 181)$$
$$= 287$$
$$\text{Area } = \sqrt{s(s-a)(s-b)(s-c)}$$
$$= \sqrt{287(287-251)(287-142)(287-181)}$$
$$= 12601.699$$
$$\approx 12602 \text{ m}^2 \text{ (correct to the nearest square metre)}$$

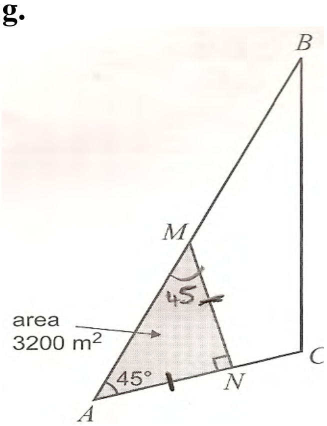
f. i. Using the Cosine rule, $BC = \sqrt{251^2 + 142^2 - (2 \times 251 \times 142\cos 45^0)}$

$$= \sqrt{32759}$$
$$= 180.996$$
$$= 181\text{m (correct to 1 decimal place).}$$

ii. Using the Sine rule, $\dfrac{181}{\sin 45^0} = \dfrac{142}{\sin\theta}$

$$\sin\theta = \frac{142\sin 45^0}{181}$$
$$\theta = \sin^{-1}\left(\frac{142\sin 45^0}{181}\right)$$
$$\theta = 33.7° \text{ (correct to 1 decimal place)}$$

g.



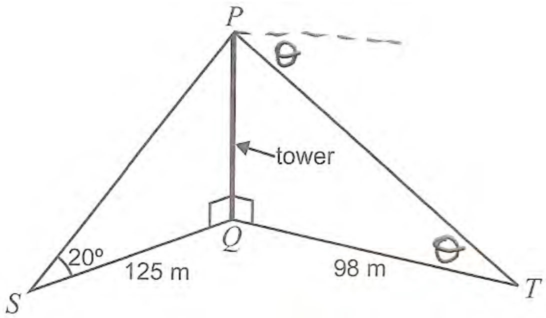
As the ΔAMN is isosceles, solve:

$$3200 = \frac{1}{2}MN^2$$

$6400 = MN^2$ and so MN is 80m.

Question 2

a.



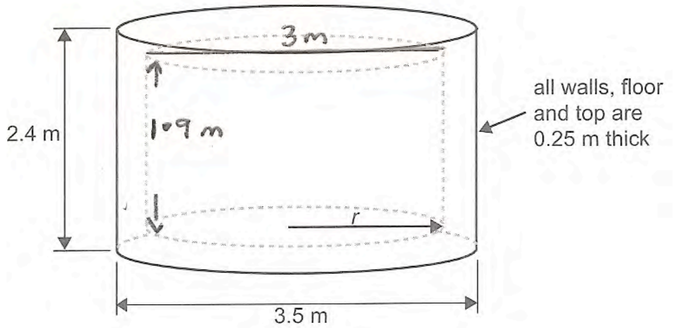
$\tan 20^0 = \dfrac{PQ}{125}$ and so $PQ = 125 \tan 20^0$

$PQ = 45.5$ m (correct to 1 decimal place)

b. The angle of depression is given by θ where $\tan\theta = \dfrac{45.5}{98}$.

$$\theta = \tan^{-1}\frac{45.5}{98} = 24.9° \text{ (correct to 1 decimal place)}$$

Question 3

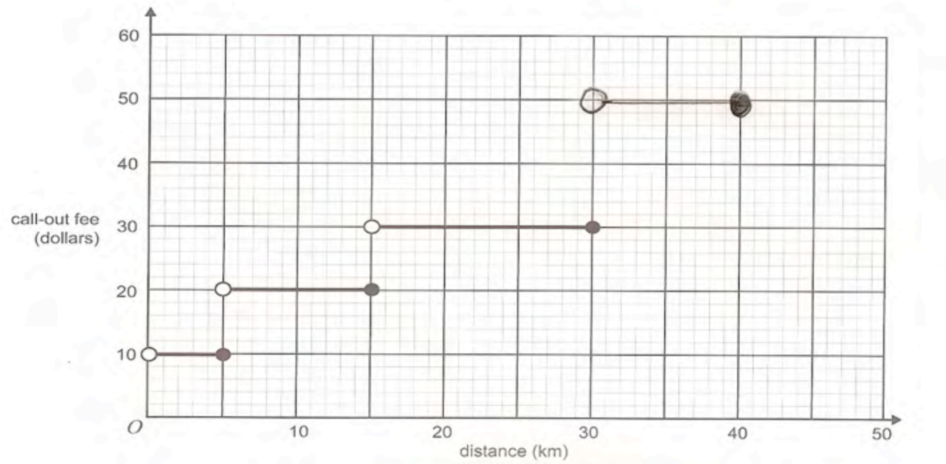


a. From the diagram, $r = \frac{3}{2} = 1.5$ m

b.

$$\begin{aligned} V_{cylinder} &= \pi r^2 h \\ &= \pi \times 1.5^2 \times (2.4 - (2 \times 0.25)) \\ &= 13.43 \\ &\approx 13 \text{ m}^3 \text{ (correct to the nearest cubic metre)} \end{aligned}$$

Module 3: Graphs and relations
Question 1

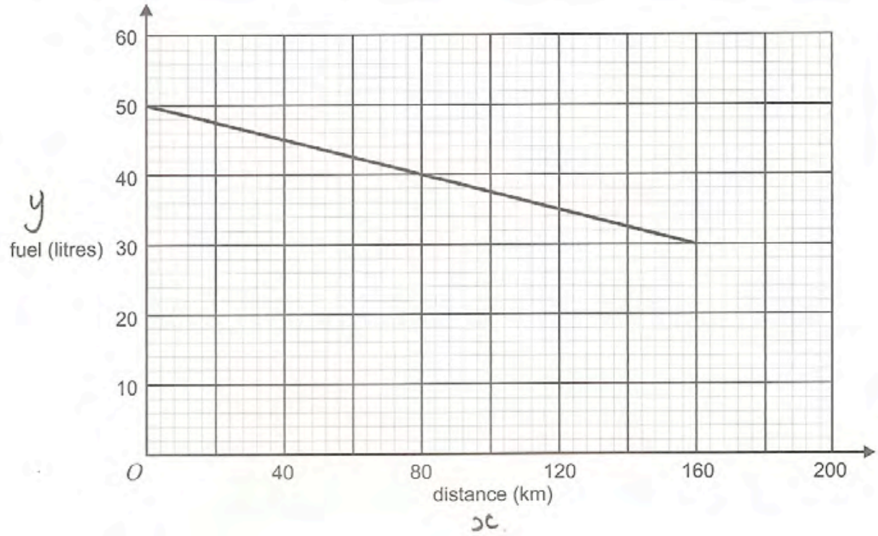


a. i. The call-out fee to travel a distance of 20 km is $30.

ii. The maximum distance travelled for a call-out fee of $10 is 5 km.

b. Refer to the diagram above for a call-out fee of $50 being charged to travel distances of more than 30 km but less than or equal to 40 km.

Question 2



a. Two coordinates on the line are (0,50) and (160,30)

Gradient $m = \dfrac{30-50}{160-0} = -\dfrac{1}{8}$

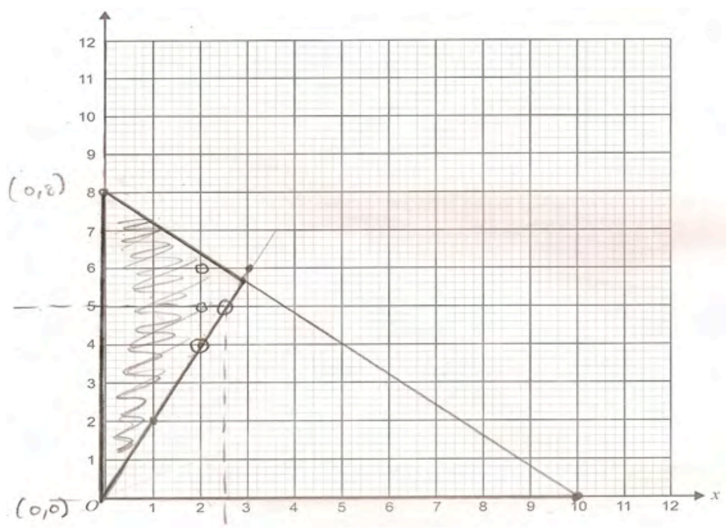
Equation of the line is $y = -\dfrac{1}{8}x + 50$

b. Solving $-\dfrac{1}{8}x + 50 = 0$, gives $x = 400$

He had already travelled 160 km and so $400 - 160 = 240$ km is the distance that he needs to travel further before the tank is empty.

c. When the tank is completely full it holds $12 + (18 \times 3.5) = 75$ litres.

Question 3



(Feasible region shaded)

a. For $20x + 25y = 200$,
The x-intercept: $20x = 200$ so $x = 10$ (10, 0)
The y-intercept $25y = 200$ so $y = 8$ (0, 8)
Refer to diagram above for the line.

b. In any one day, the number of dogs clipped (y) is at least twice the number of dogs washed (x).
Inequality 4 is $y \geq 2x$

c. i. Refer to the diagram above for the boundaries of the region represented by Inequalities 1 to 4.
ii. From the graph, when $y = 5$, $x = 2.5$.
So the maximum number of dogs that could be washed is 2.

d. The profit from washing one dog is \$40 and the profit from clipping one dog is \$30 so the total profit is given by the equation: $P = 40x + 30y$

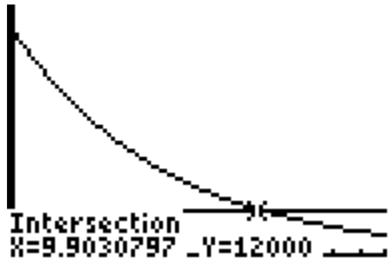
e. i. Using the profit equation $P = 40x + 30y$ and the feasible region coordinates:
(0, 8) $P = 0 \times 40 + 8 \times 30 = \$240$
(2, 4) $P = 2 \times 40 + 4 \times 30 = \$200$
(2, 5) $P = 2 \times 40 + 5 \times 30 = \$230$
(2, 6) $P = 2 \times 40 + 6 \times 30 = \$260$
Maximum total profit is achieved if 2 dogs are washed and 6 dogs are clipped.

ii. Maximum total profit is \$260.

Module 4: Business-related mathematics
Question 1

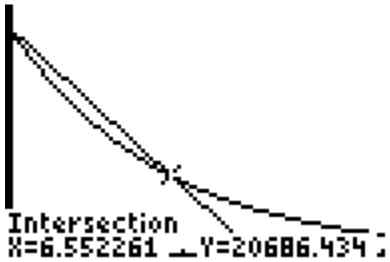
a **i.** Annual depreciation = $60000 \times 0.1 = \$6000$
ii. After three years, the value of the machine will be $60000 - (6000 \times 3) = \$42000$
iii. $12000 = 60000 - 6000n$
$6000n = 48000$ so $n = 8$
The machine is worth \$12000 after 8 years.

b. $V = 60000 \times (0.85)^n$.
i. Annual depreciation = $1 - 0.85 = 0.15 = 15\%$
ii. When $n = 3$, $V = 60000 \times 0.85^3 = \$36847.50$.
The machine is worth \$36847.50
iii. Solve $12000 = 60000 \times 0.85^n$



The machine will first fall below \$12000 at the end of 10 years.

c. Solve $60000 - 6000n = 60000 \times 0.85^n$



The machine will be less using Flat Rate rather than Reducing Balance Depreciation at the end of 7 years.

Question 2

Cost of a new machine = $60000 \times (1.02)^8$ = \$70300 (correct to the nearest dollar)

Question 3

a. P = \$7000 , r = 6.25% , t = 8

$$I = \frac{7000 \times 6.25 \times 8}{100} = \$3500$$

Investment = Principal + Interest = \$7000 + \$3500 = \$10500

b. P =\$10000, r = 6% p.a., t = 8 years but compounded **quarterly.**

$$A = 10000 \times (1 + \frac{6}{400})^{32}$$
$$= \$16103.24$$
$$\approx \$16103$$

After 8 years the investment is worth \$16103 (correct to the nearest dollar)

c.

```
 N=96
 I%=6.5
 PV=500
 PMT=200
▪FV=-25935.30411
 P/Y=12
 C/Y=12
 PMT:END BEGIN
```

After 8 years, the investment is worth \$25935 (correct to the nearest dollar)

Question 4

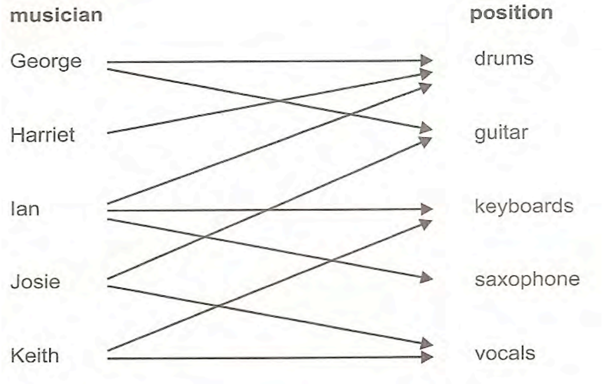
```
 N=24
 I%=10
 PV=20000
▪PMT=-922.89852…
 FV=0
 P/Y=12
 C/Y=12
 PMT:END BEGIN
```

The monthly repayment is \$922.90.

The total amount of interest:

$= (24 \times 922.90) - 20000 = \$2150$ (correct to the nearest dollar)

Module 5: Networks and decision mathematics
Question 1



a George must play the guitar.

b.

Person	Position
Harriet	Drums
Ian	Saxophone
Keith	Keyboards

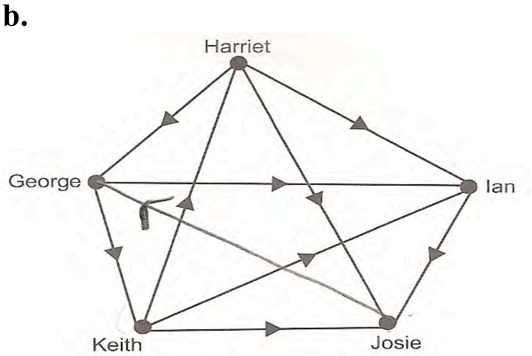
Question 2
a.

Matrix 1

$$\begin{array}{cc} & \textbf{loser} \\ & \begin{array}{ccccc} G & H & I & J & K \end{array} \\ \textbf{winner} \begin{array}{c} G \\ H \\ I \\ J \\ K \end{array} & \begin{bmatrix} \mathbf{0} & 0 & 1 & 0 & 1 \\ 1 & \mathbf{0} & 1 & 1 & 0 \\ 0 & 0 & \mathbf{0} & 1 & 0 \\ 1 & 0 & 0 & \mathbf{0} & 0 \\ 0 & 1 & 1 & 1 & \mathbf{0} \end{bmatrix} \end{array}$$

The figures in bold in Matrix 1 are all zero as no musician can compete against his/herself.

b.



Missing edge – Josie defeats George.

c. Matrix 2 shows the 2 step dominances.

Matrix 2

$$\begin{array}{cc} & \begin{array}{ccccc} G & H & I & J & K \end{array} \\ \begin{array}{c} G \\ H \\ I \\ J \\ K \end{array} & \begin{bmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 2 & 0 & 1 & x & 0 \end{bmatrix} \end{array}$$

George defeated Keith and Keith defeated Ian so George has a 2 step dominance over Ian.

d. $x = 2$

e.

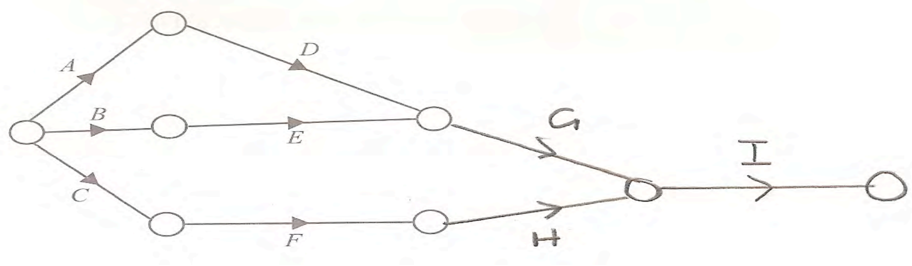
Musician	**Dominance value (one step)**	**Dominance value (two step)**	**Total**
George	2	4	6
Harriet	3	4	7
Ian	1	1	2
Josie	1	2	3
Keith	3	5	8

Keith came first and Ian came last.

Question 3

Activity	Immediate predecessors
A	–
B	–
C	–
D	A
E	B
F	C
G	D, E
H	F
I	G, H

The network for the activities above is:



b. There are 5 non-critical activities: A, C, D, F and H.

c. The critical path for the project is B E G I.

d. Duration of activity I = $19 - 12 = 7$ hours

e. C, F and H cannot add to more than the latest starting time for activity I which is 12. Time for F&H=$12 - C = 12 - 3 = 9$ hours. As activity C has a float time of 1 hour, the time for F&H can be no more than 8 hours.

Module 6: Matrices
Question 1

a. The order of Q is 2×3.

b. i. M = QP = $\begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix} \begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix} = \begin{bmatrix} 145978 \\ 171848.50 \end{bmatrix}$

ii. Matrix M gives the total revenue from selling products A, B and C at Eastown and Noxland.

c. Order of $\begin{bmatrix} 14.50 \\ 21.60 \\ 19.20 \end{bmatrix}$ is (3 × 2) and order of $\begin{bmatrix} 2500 & 3400 & 1890 \\ 1765 & 4588 & 2456 \end{bmatrix}$ is (2 × 3)

PQ is not defined as the number of columns of P $\neq$ number of rows in Q.

Question 2

a. The transition matrix T is:

$$\begin{array}{cc} & \textbf{This week} \\ & \begin{array}{ccc} S & E & N \end{array} \\ T = & \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \end{array} \begin{array}{l} S \\ E \ \textbf{Next week} \\ N \end{array}$$

b. $K_0 = \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix}$

c. $K_1 = T \times K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 268800 \\ 136200 \\ 195000 \end{bmatrix}$

d. $$K_{38} = T^{38} \times K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{38} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

$$K_{50} = T^{50} \times K_0 = \begin{bmatrix} 0.80 & 0.09 & 0.10 \\ 0.12 & 0.76 & 0.05 \\ 0.08 & 0.15 & 0.85 \end{bmatrix}^{50} \begin{bmatrix} 300000 \\ 120000 \\ 180000 \end{bmatrix} = \begin{bmatrix} 194983 \\ 150513 \\ 254504 \end{bmatrix}$$

Question 3

a. $$\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix}$$

b. Using a graphing calculator the determinant of the matrix $\begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} = \mathbf{A}$ is

```
det([A])
                    1
```

So the equations have a unique solution because the determinant is non-zero.

c. Using a graphing calculator, the inverse matrix of A is

```
[A]-1
    [[-1  3   2 ]
     [1   -2  -1]
     [2   -4  -3]]
```

d. $\begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & 1 \\ 2 & -4 & -3 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 1 & -1 & 1 \\ 0 & 2 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 & 3 & 2 \\ 1 & -2 & 1 \\ 2 & -4 & -3 \end{bmatrix} \begin{bmatrix} 12 \\ 1 \\ 6 \end{bmatrix}$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 2 \end{bmatrix}$$

The ideal number is 3 bookshops, 4 sports shoe shops and 2 music stores.